New asymmetric gravity-capillary and flexural waves

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ICERM , April 2017

COWORKERS

Zhan Wang

Tao Gao

Paul Milewski

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- inviscid, incompressible, irrotational
- gravity
- surface tension
- steady

NON SYMMETRIC WAVES....in two and three dimensions

- Periodic waves
- Solitary waves
- Generalised Solitary waves

flexural waves (thursday.....Olga....).

stability

PART 1

TWO-DIMENSIONAL FLOWS

FORMULATION

GRAVITY-CAPILLARY WAVES

$$\phi_{xx} + \phi_{yy} = 0$$

$$\phi_y = \phi_x \zeta_x \quad \text{on} \quad y = \zeta(x)$$

$$\frac{1}{2}(\phi_x^2 + \phi_y^2) + gy - \frac{T}{\rho}\kappa = B \quad \text{on} \quad y = \zeta(x)$$

$$\phi_y = 0 \quad \text{on} \quad y = -h$$

FLEXURAL WAVES

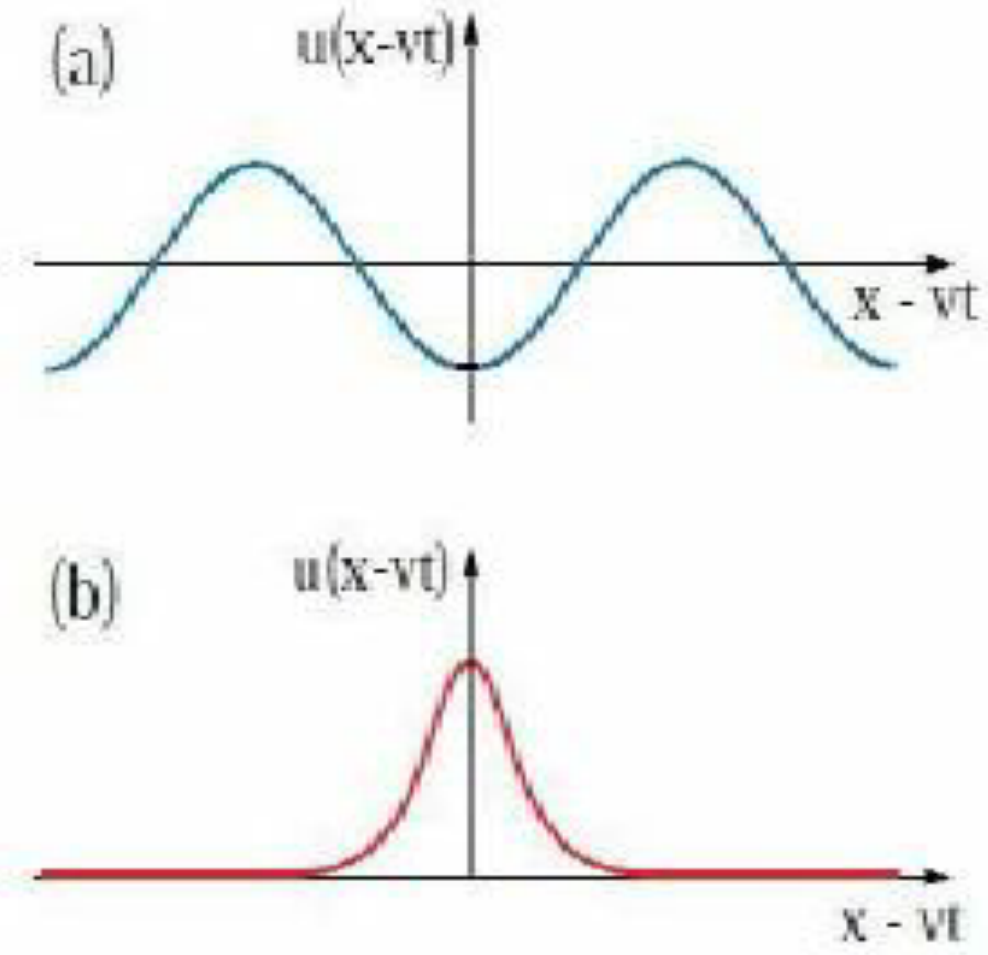
$$\frac{D}{\rho}(\partial_s^2 \kappa + \frac{1}{2}\kappa^3)$$

$T =$ surface tension, $D =$ flexural rigidity

$$\kappa = \frac{\zeta_{xx}}{(1 + \zeta_x^2)^{3/2}}$$

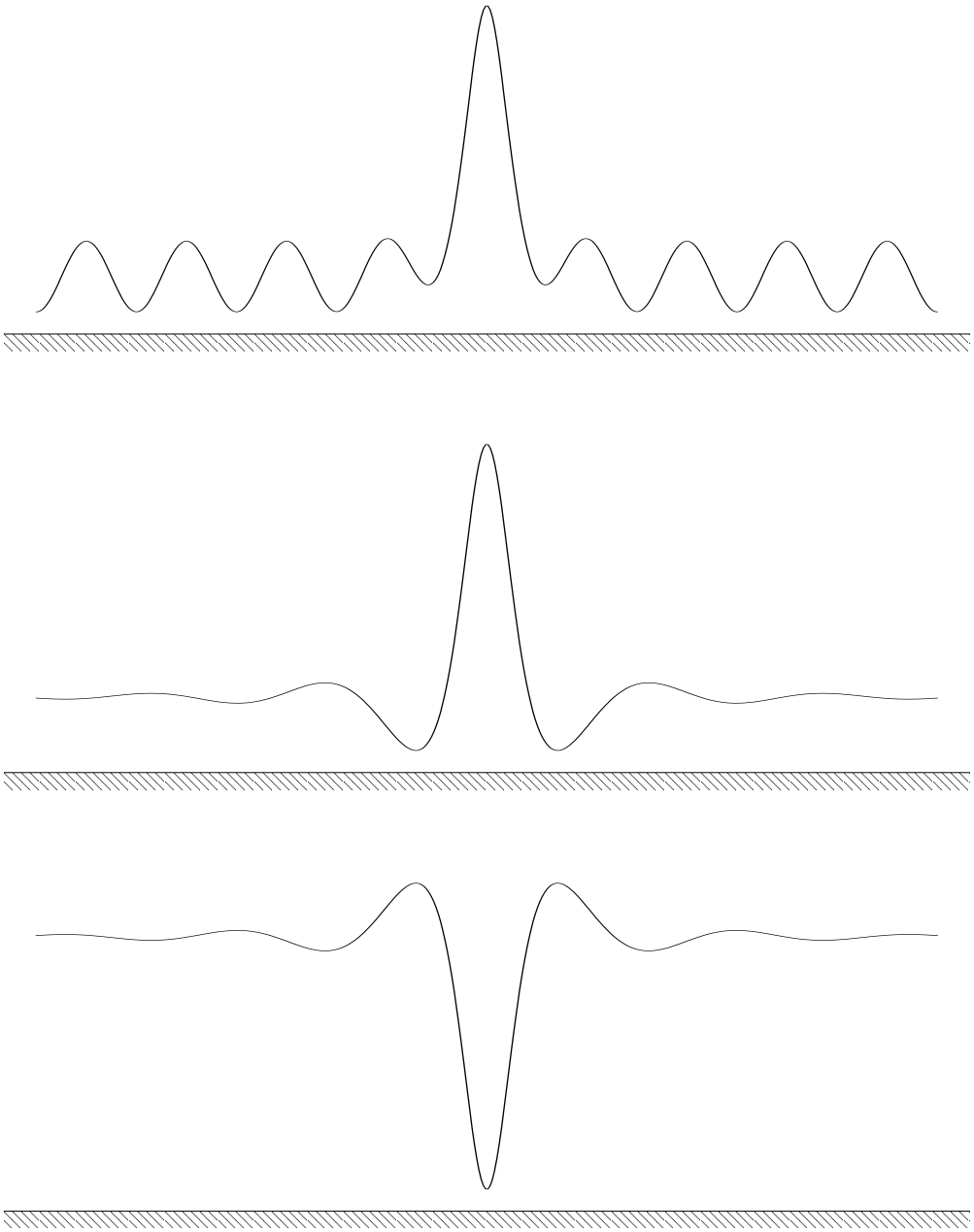
PERIODIC and SOLITARY waves

Gravity waves



Craig W. and Sternberg P. (1988)

Gravity-capillary solitary waves



NUMERICAL METHODS

boundary integral equation methods, series truncation methods or ANY OTHER METHODS....

1. Iterations by using Newton's method

2. Continuation methods

3. INITIAL GUESS: bifurcations, symmetry breaking...

Dimensioless variables: $(\frac{T}{\rho g})^{1/2}$ (reference length), $(\frac{T}{\rho g^3})^{1/2}$ (reference time)

amplitude: A

phase velocity: c

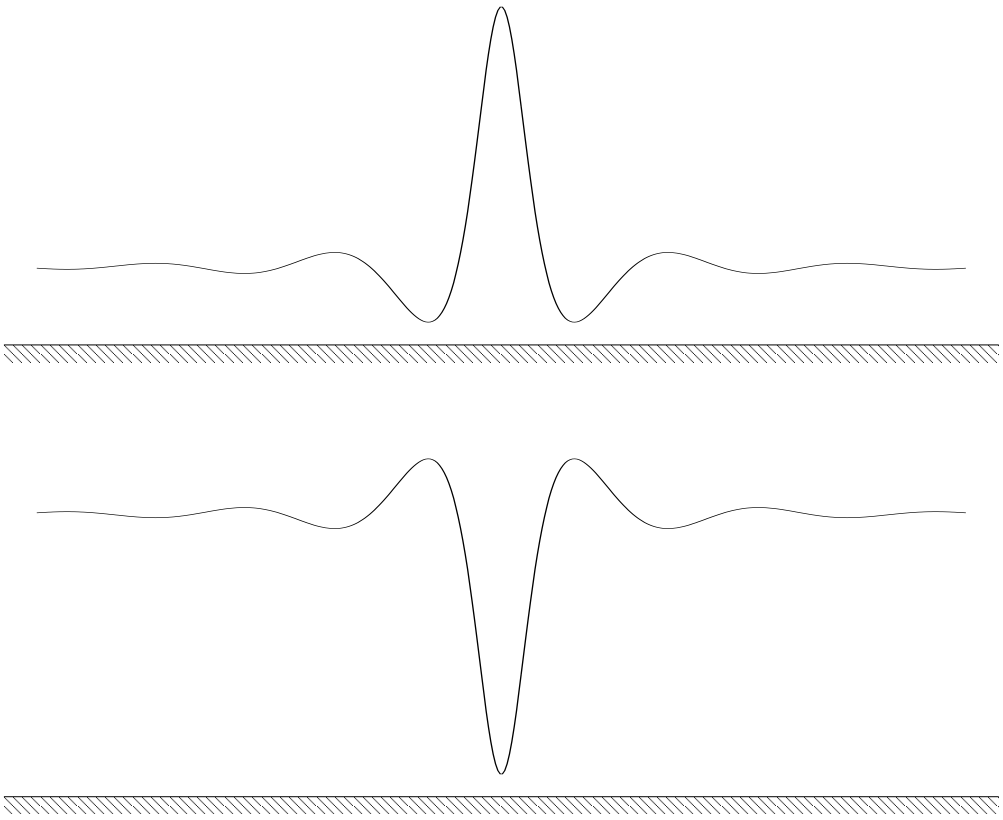
energy: E

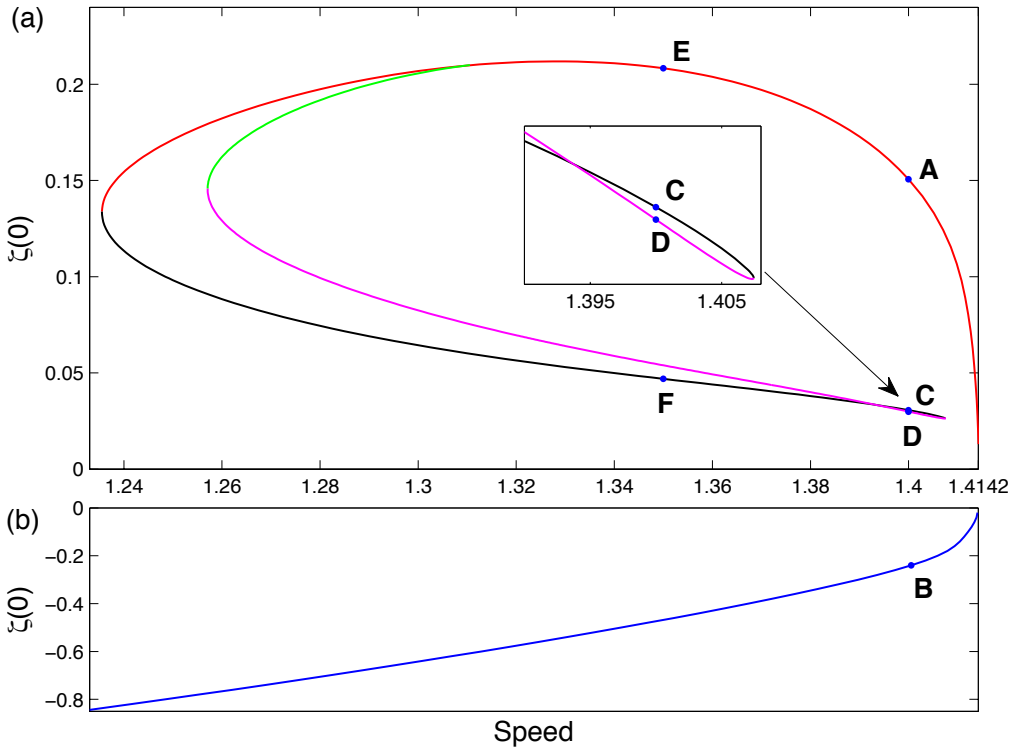
$$E = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\eta}(\phi_x^2 + \phi_y^2)dydx + \frac{1}{2}\int_{-\infty}^{\infty}\eta^2 dx$$
$$+ \int_{-\infty}^{\infty}(\sqrt{1+\eta_x^2} - 1)dx$$

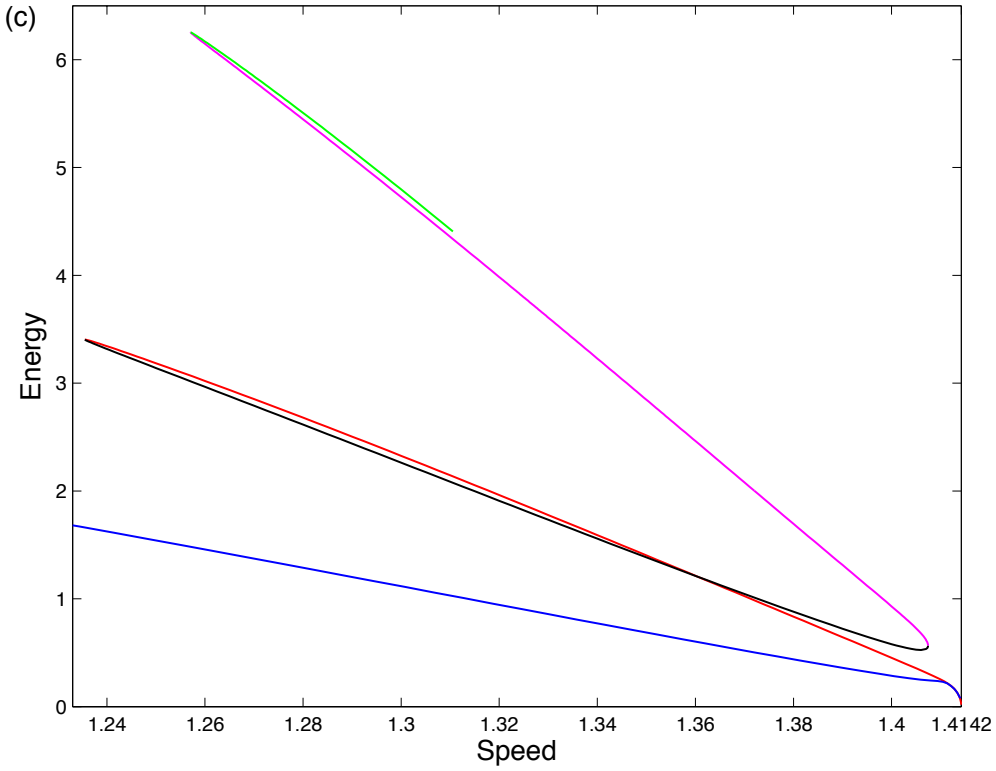
Boundary integral equation, Newton iterations, continuation

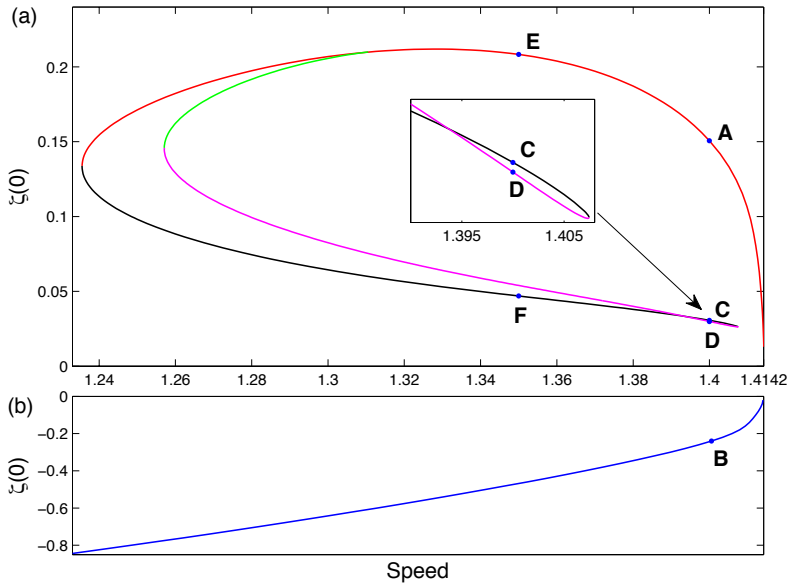
Gravity capillary solitary waves

infinite depth

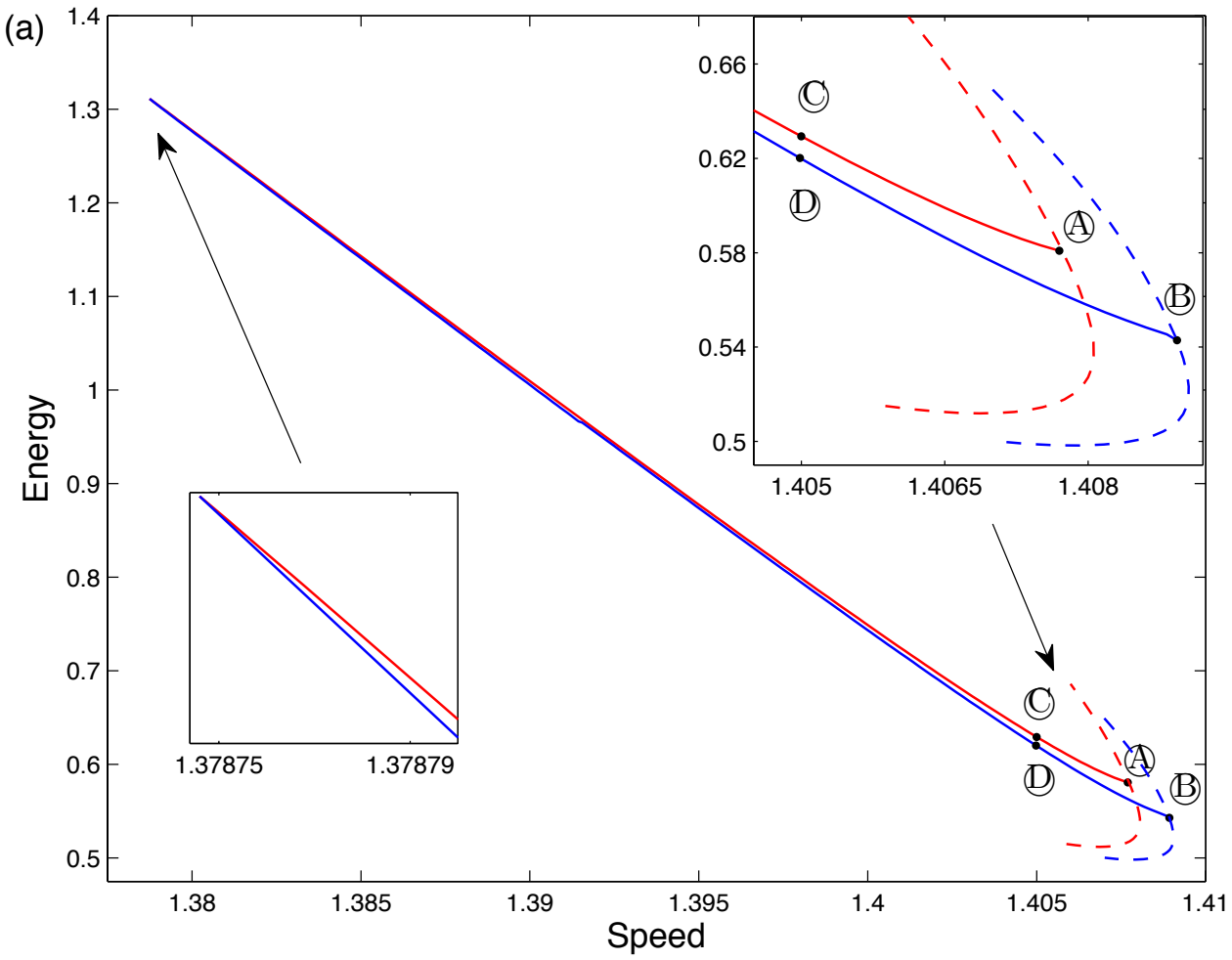


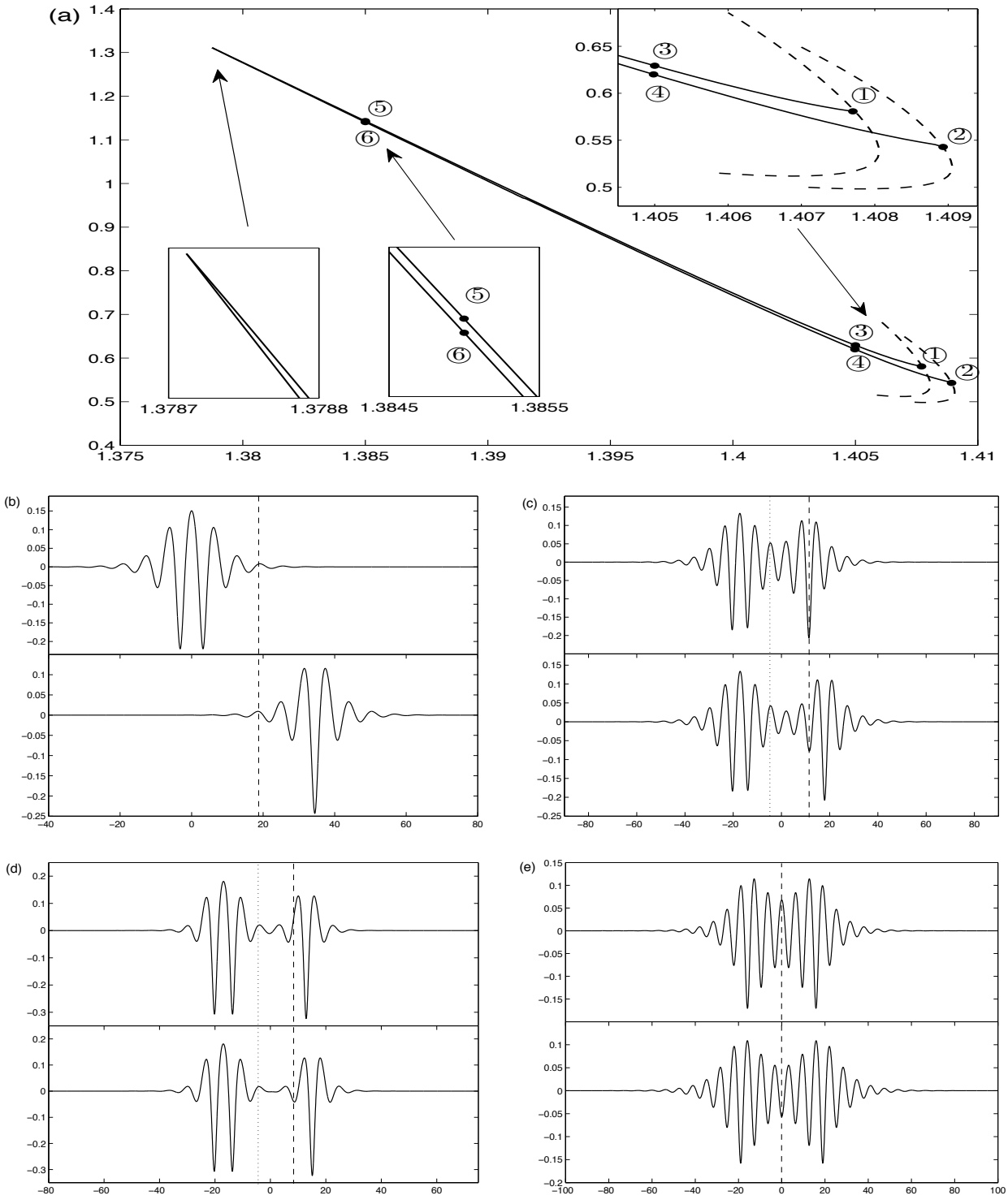

(a)
E
A
C
D
F
C
D
1.395
1.405
ζ(0)
0.2
0.15
0.1
0.05
0
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.4142
(b)
B
ζ(0)
0
−0.2
−0.4
−0.6
−0.8
Speed

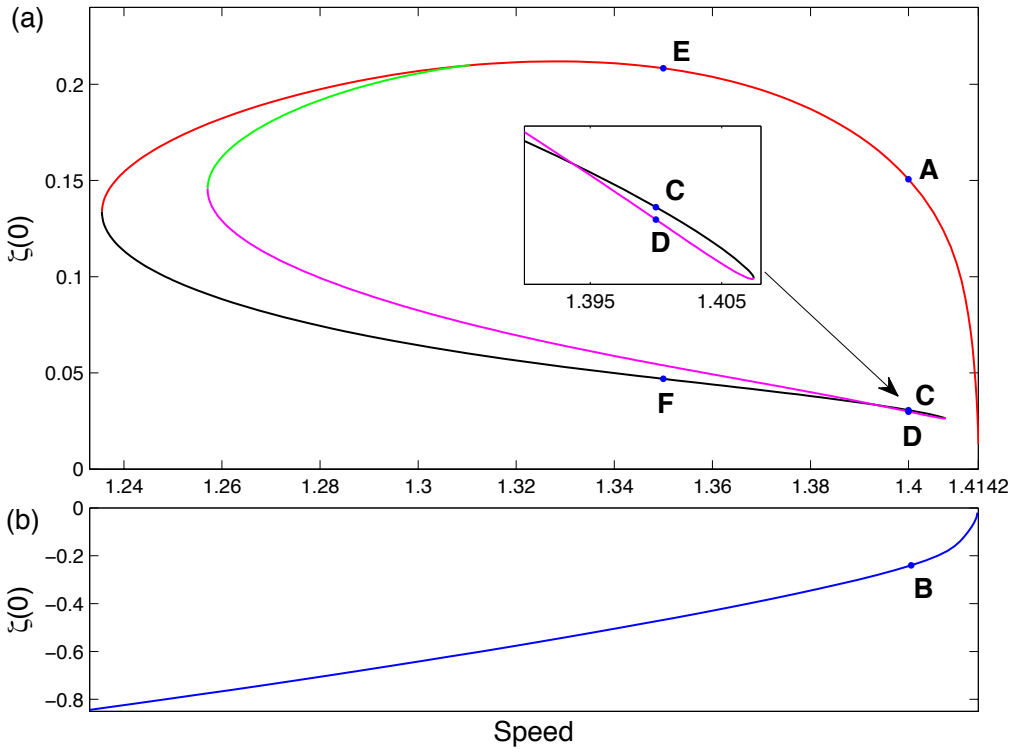

(c)
Energy
Speed
0
1
2
3
4
5
6
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.4142

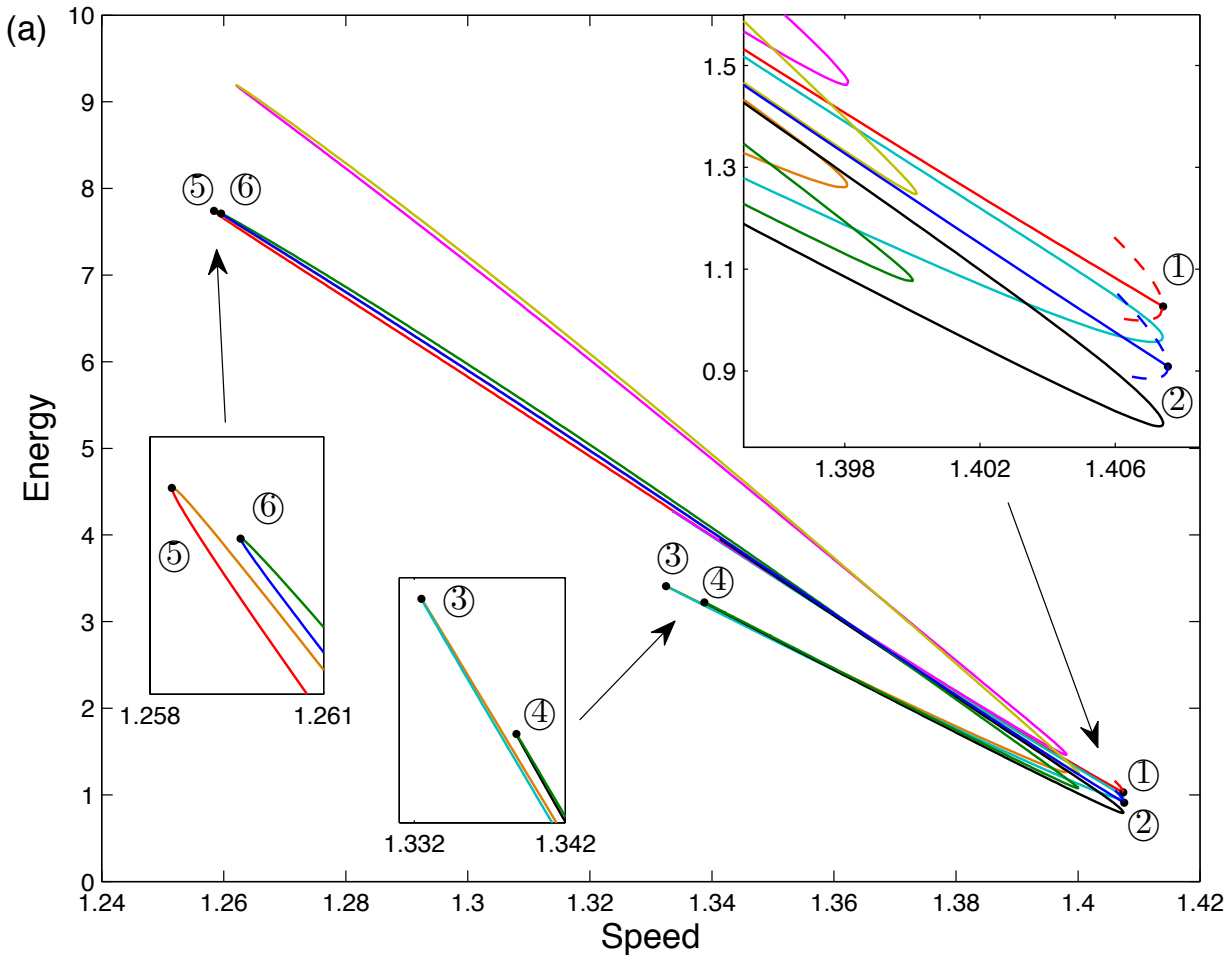


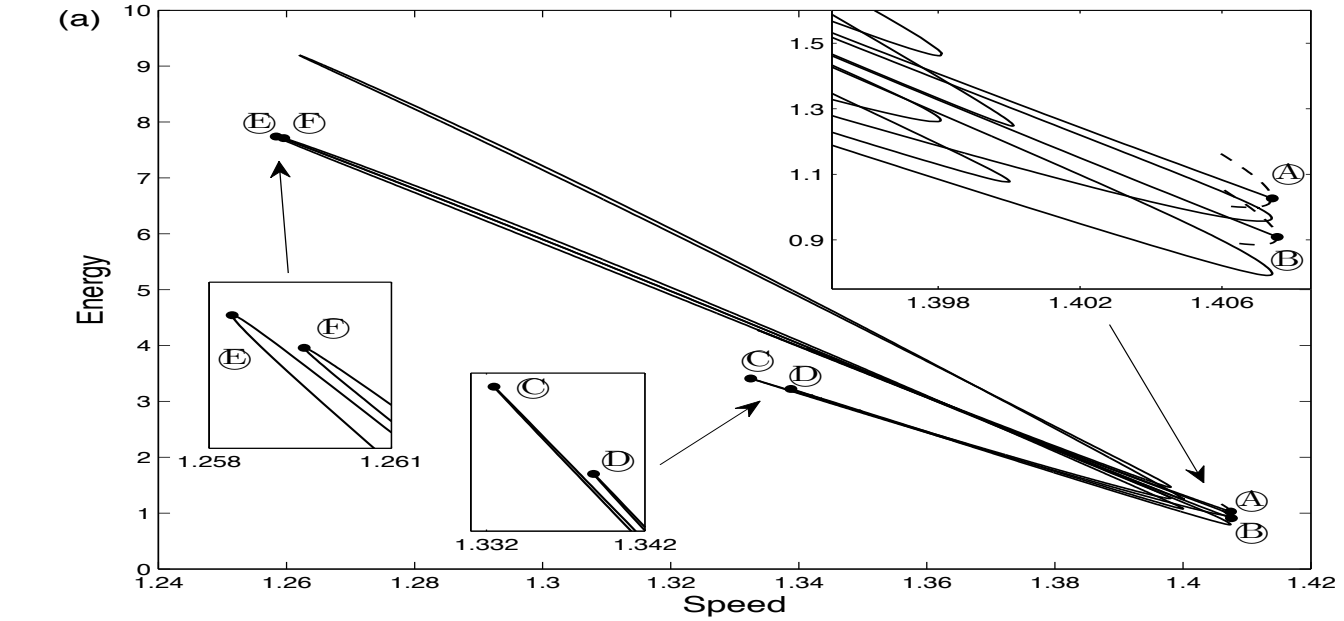
Zufiria (1987), Buffoni, Champneys and Toland (1996), Yang and Akylas (1997), Champneys and Groves (1997).....

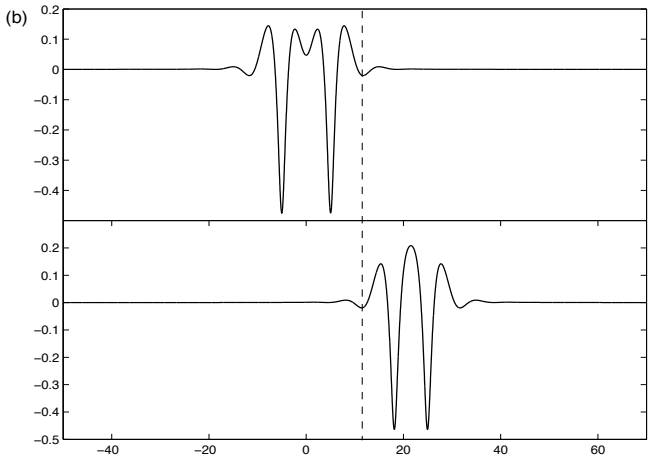

(a)
Energy
Speed
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
1.38
1.385
1.39
1.395
1.4
1.405
1.41
0.66
0.62
0.58
0.54
0.5
1.405
1.4065
1.408
1.37875
1.37879
C
D
A
B

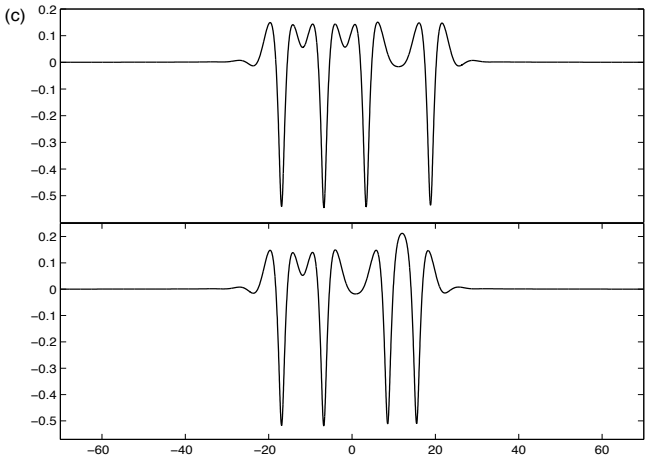

(a)
(b)
(c)
(d)
(e)
1.3787
1.3788
1.3845
1.3855

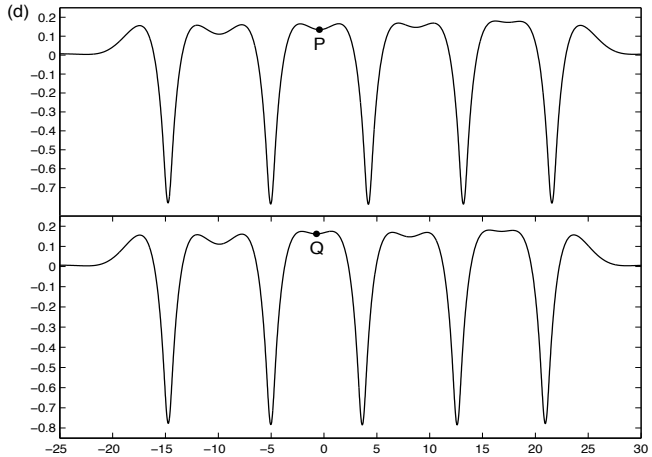

(a)
E
A
C
D
F
C
D
1.395
1.405
0.2
0.15
0.1
0.05
0
ζ(0)
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.4142
(b)
B
0
−0.2
−0.4
−0.6
−0.8
ζ(0)
Speed

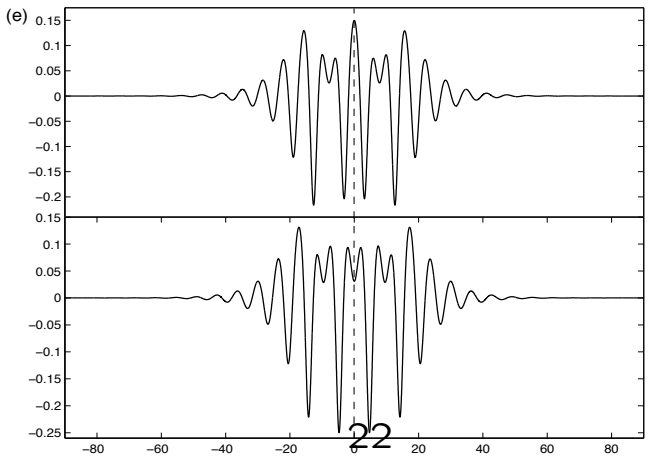

(a)
Energy
Speed
10
9
8
7
6
5
4
3
2
1
0
1.24
1.26
1.28
1.3
1.32
1.34
1.36
1.38
1.4
1.42
①
②
③
④
⑤
⑥
1.5
1.3
1.1
0.9
1.398
1.402
1.406
1.258
1.261
1.332
1.342


(a)
Energy
Speed
E
F
C
D
A
B
1.258
1.261
1.332
1.342
1.398
1.402
1.406


(b)

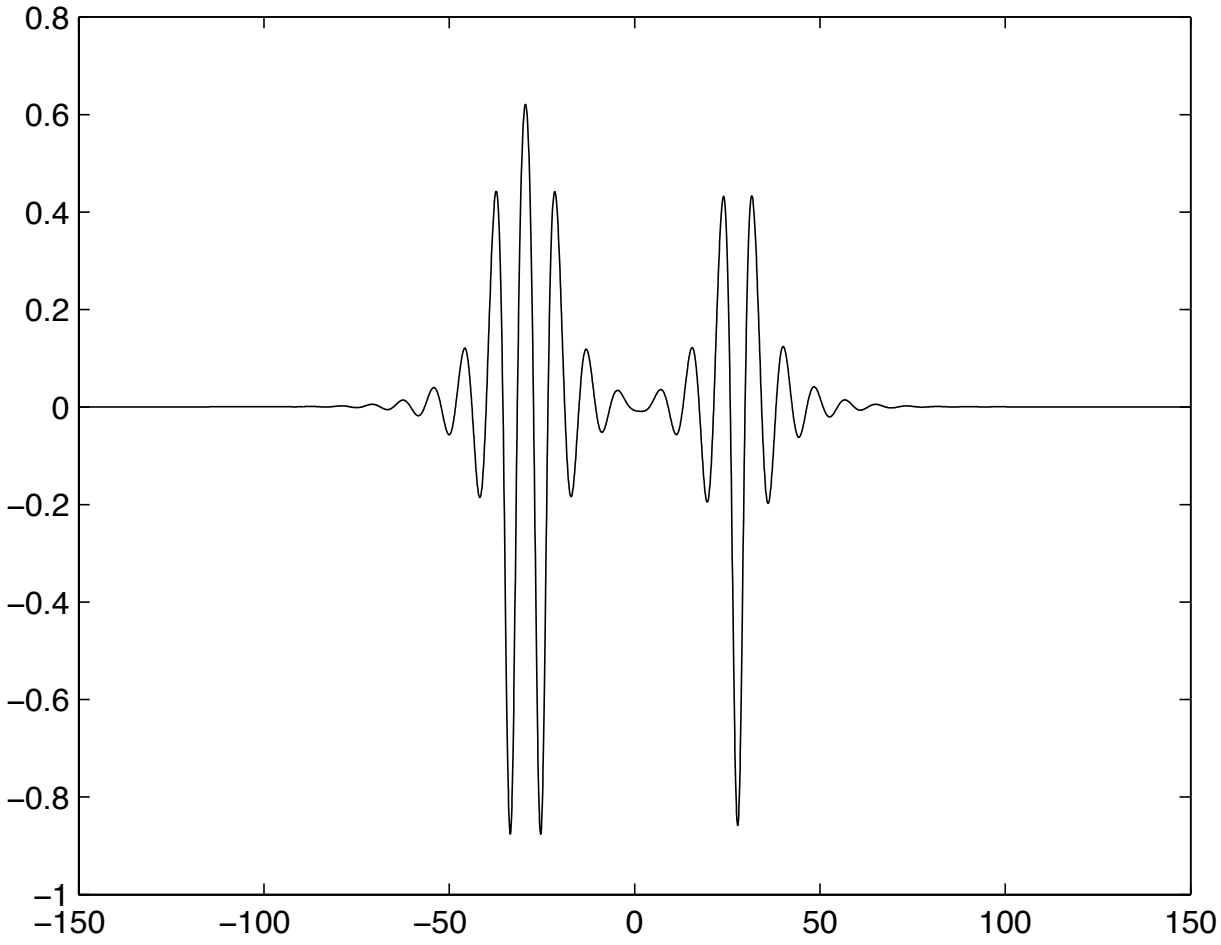

(c)


(d)
P
Q

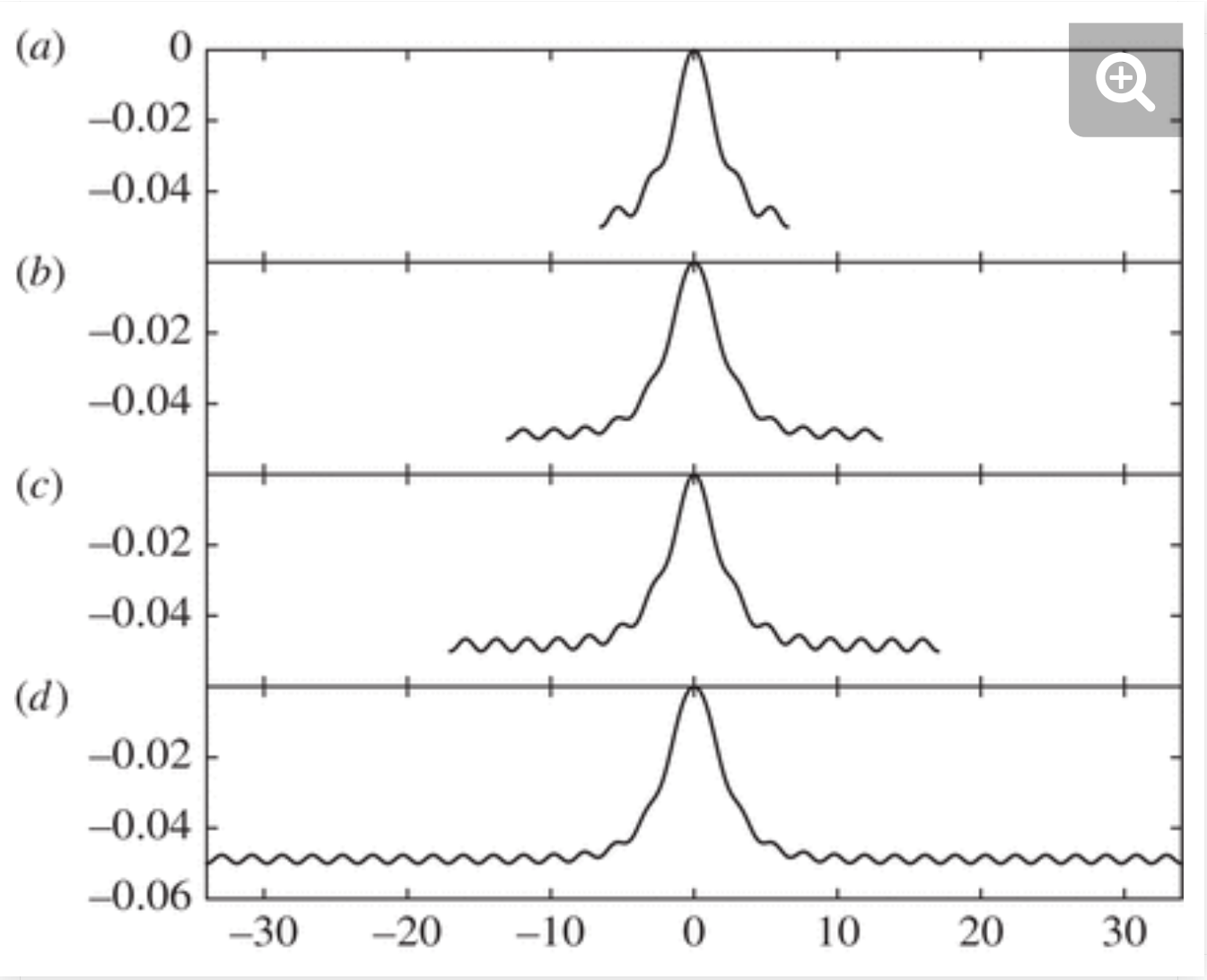

(e)

HYDROELASTIC WAVES

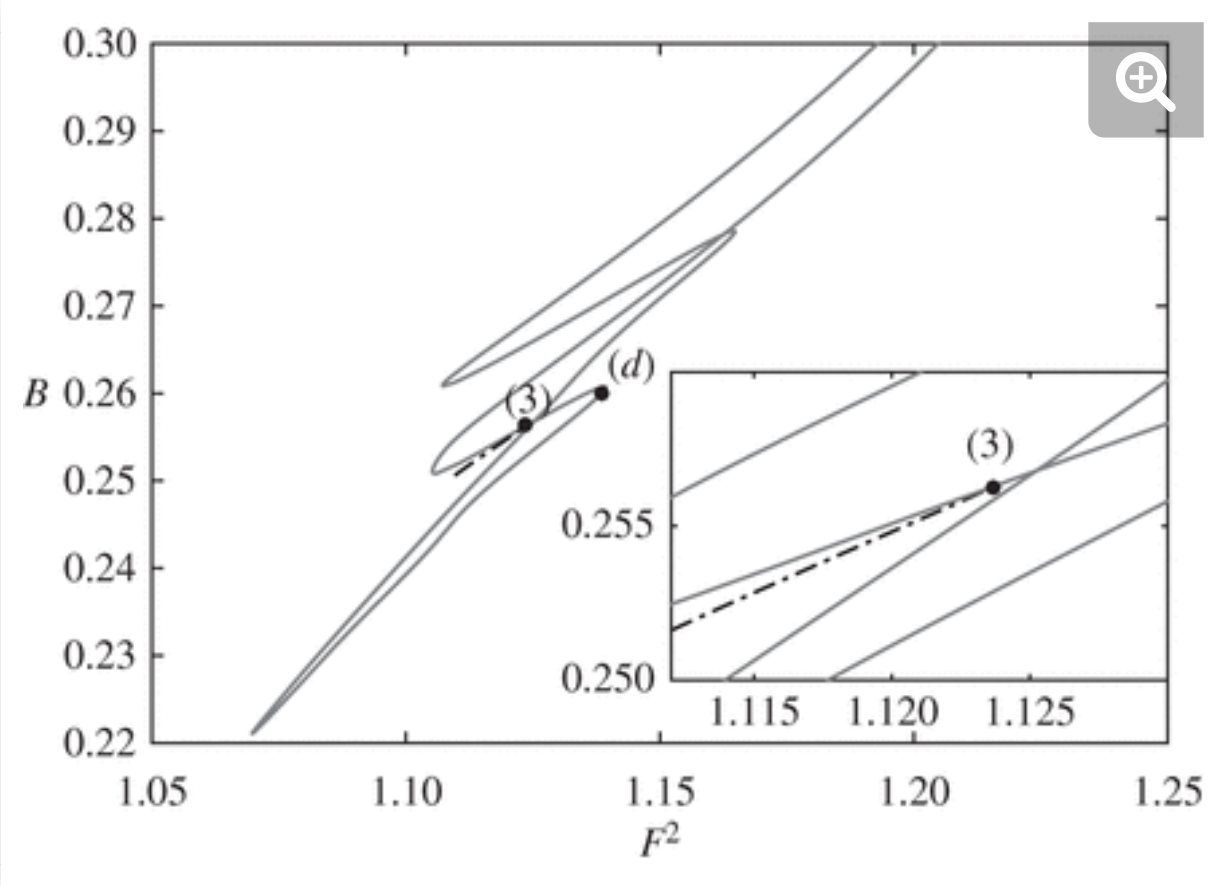
Tao Gao, Zhan Wang

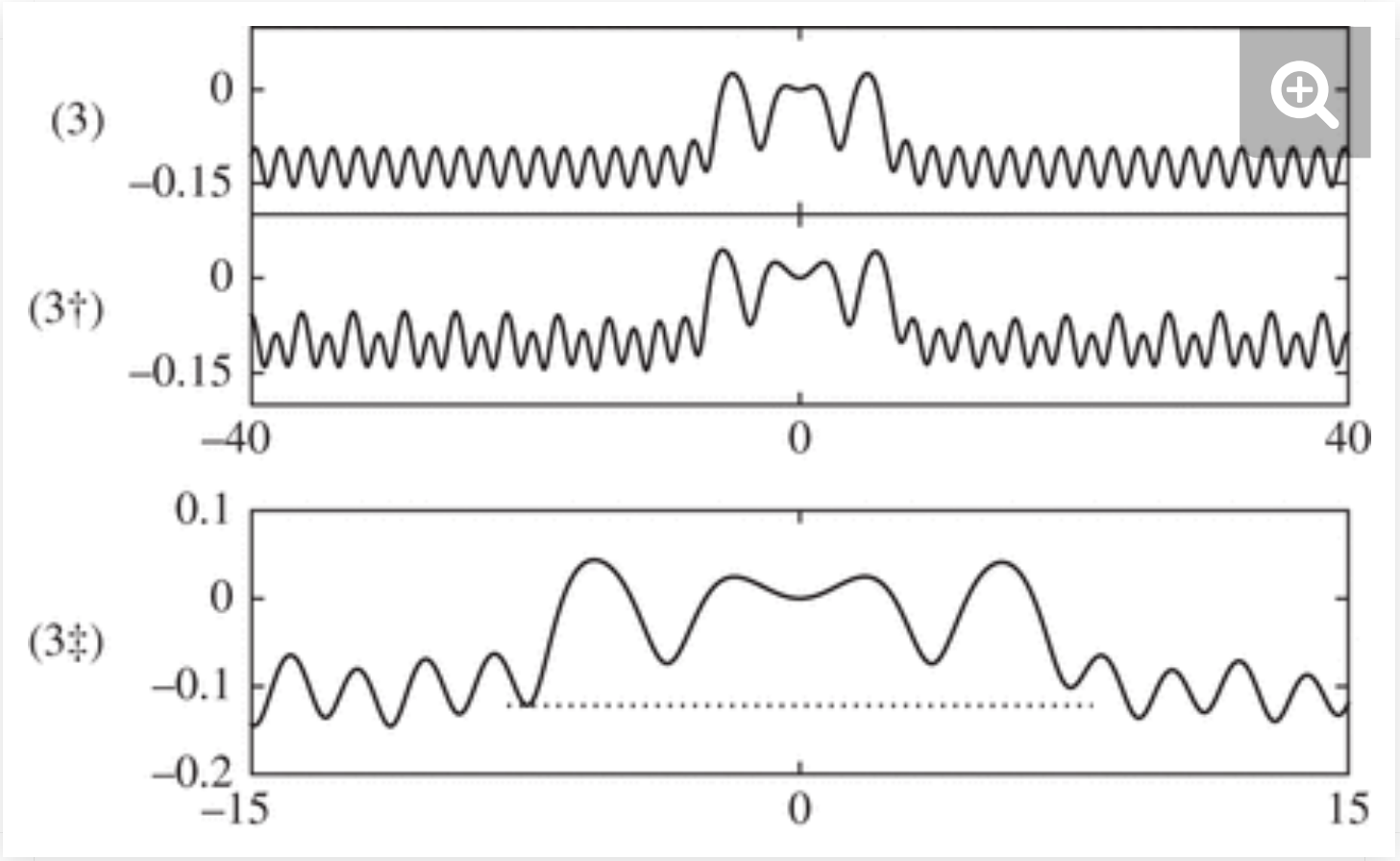

0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−150
−100
−50
0
50
100
150

GENERALISED SOLITARY WAVES

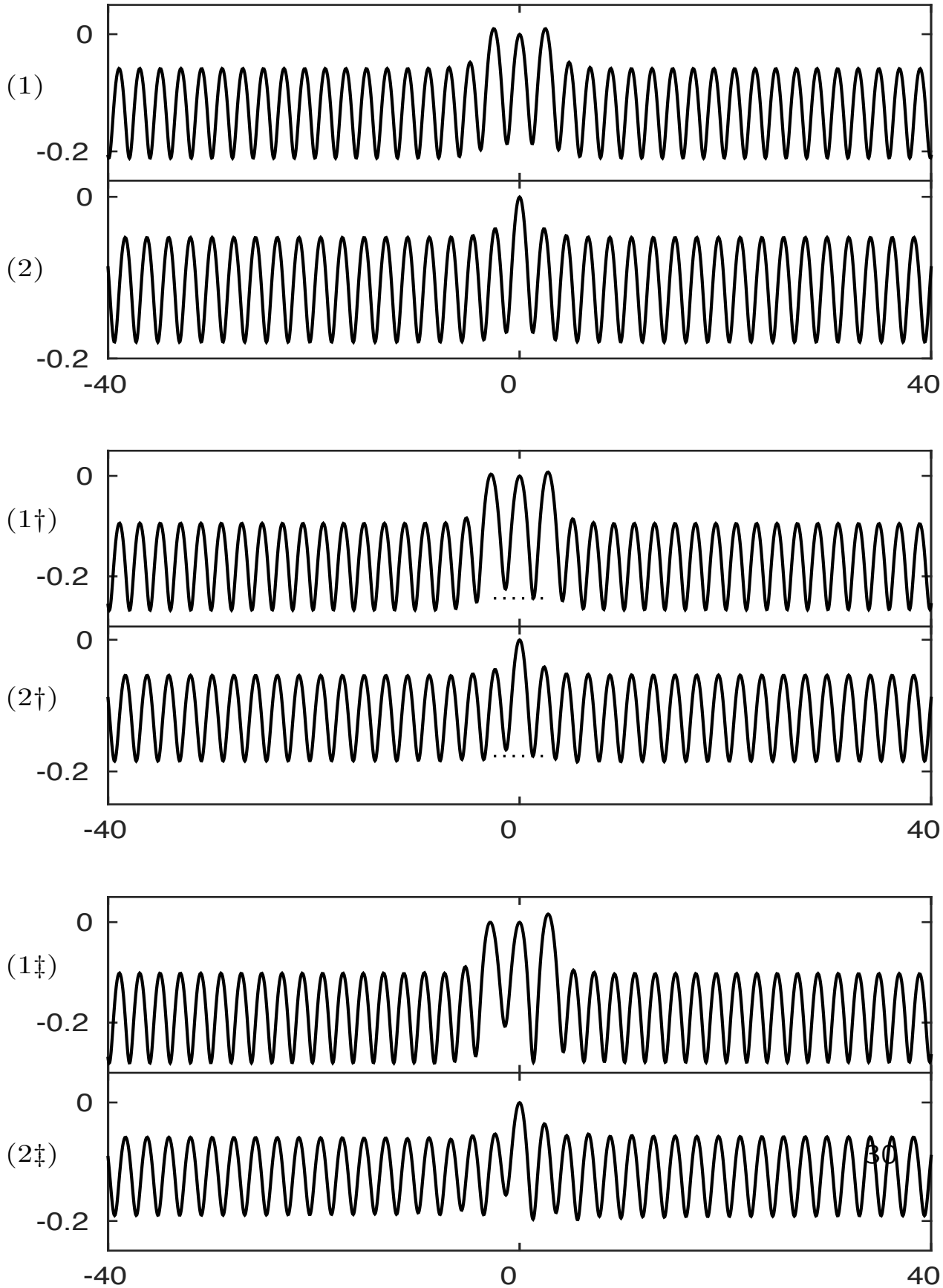

(a)
0
–0.02
–0.04
(b)
–0.02
–0.04
(c)
–0.02
–0.04
(d)
–0.02
–0.04
–0.06
–30
–20
–10
0
10
20
30

Clamond et al, Journal of Fluid Mechanics (2015) 784, pp 664-680


0.30
0.29
0.28
0.27
B 0.26
0.25
0.24
0.23
0.22
1.05
1.10
1.15
1.20
1.25
F^2
(3)
(d)
0.255
0.250
1.115
1.120
1.125
(3)



Wang Z., Parau E.I. , Milewski P.A. and Vdb (2014) Proc. Roy. Soc. A 470


(1)
0
-0.2
(2)
0
-0.2
-40
0
40
(1†)
0
-0.2
(2†)
0
-0.2
-40
0
40
(1‡)
0
-0.2
(2‡)
0
-0.2
-40
0
40

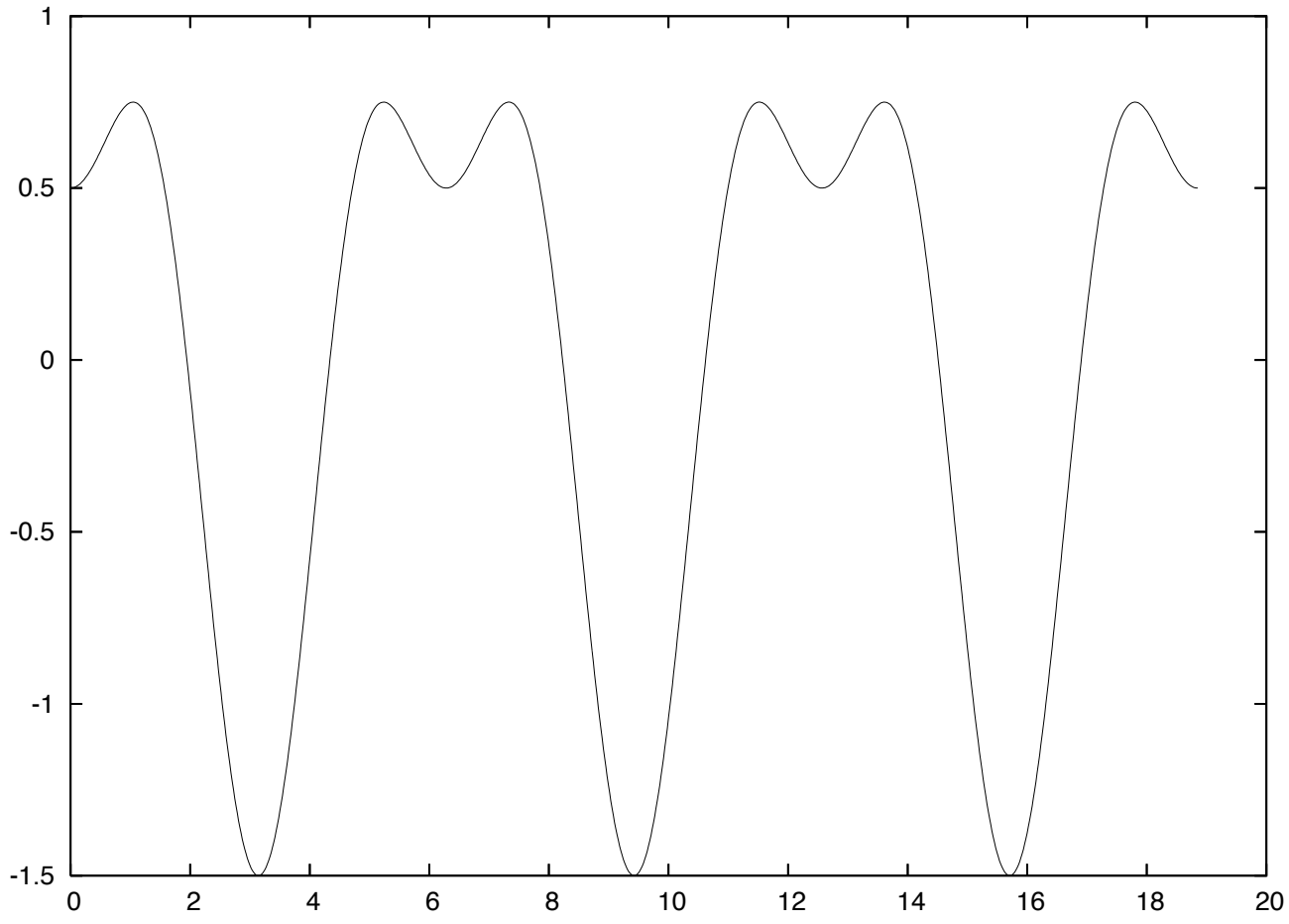
Non-symmetric PERIODIC gravity-capillary waves

Tao Gao and Zhan Wang

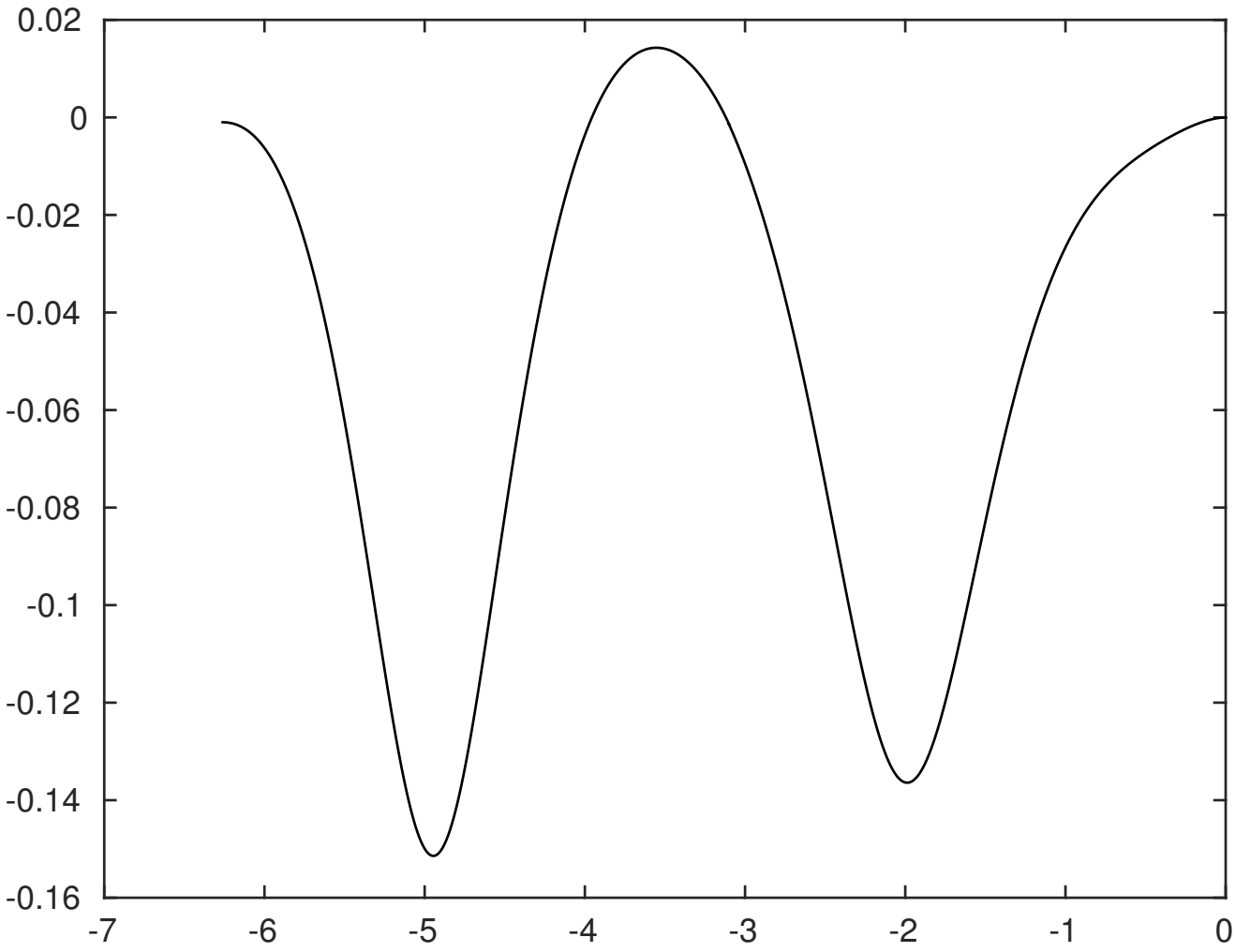
Zufiria (1987)

Shimizu ans Shoji (2012)

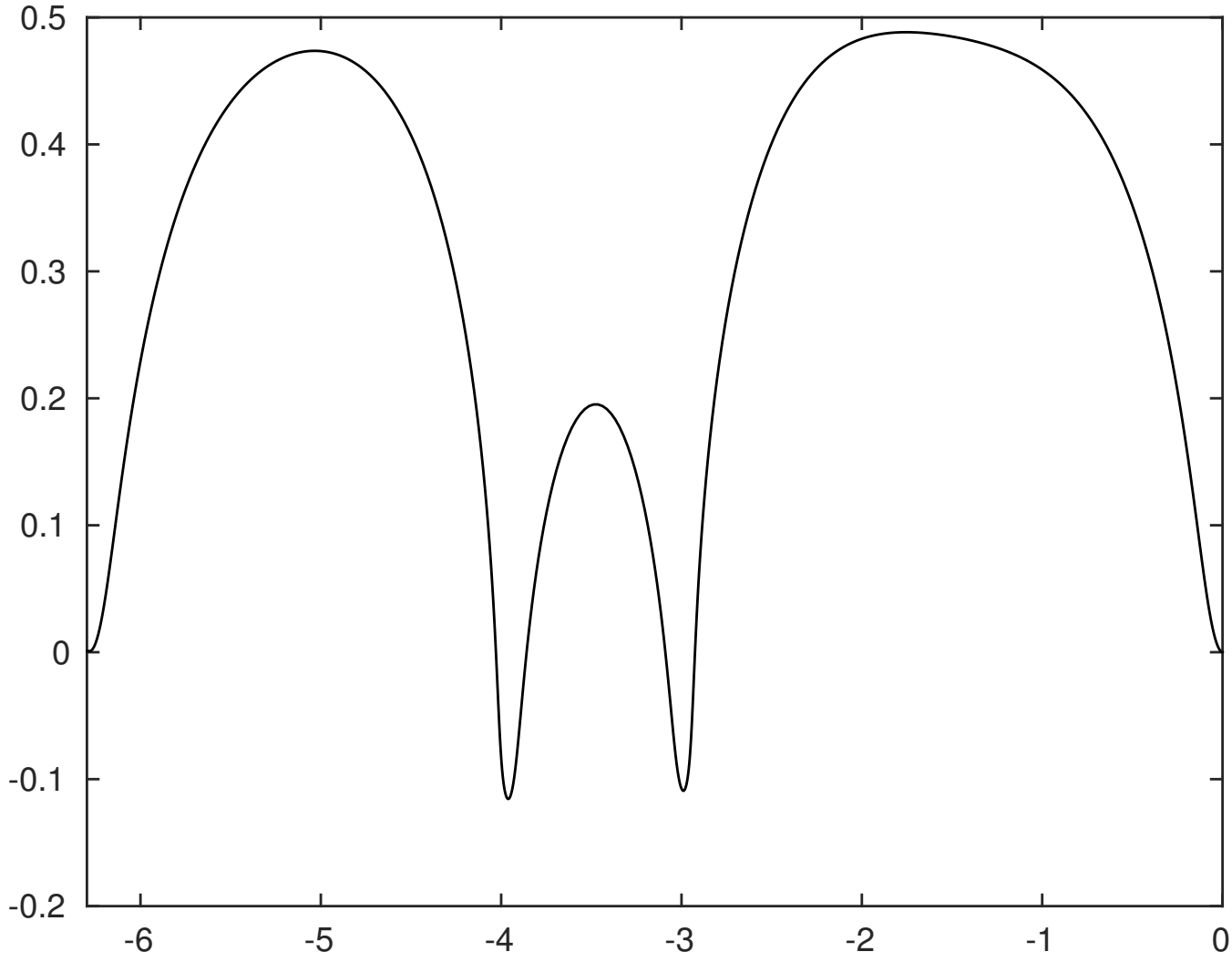
Symmetric waves



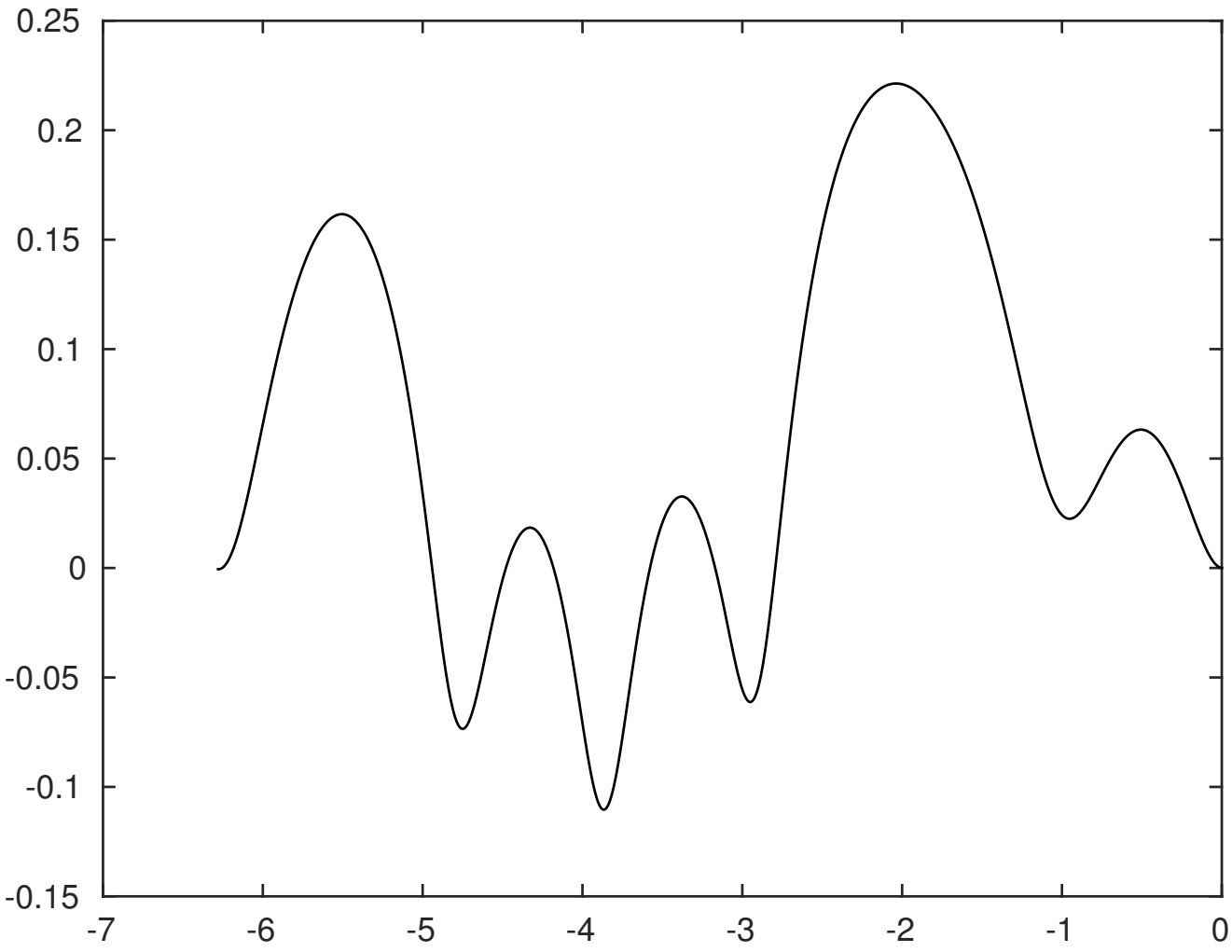
Non-symmetric waves



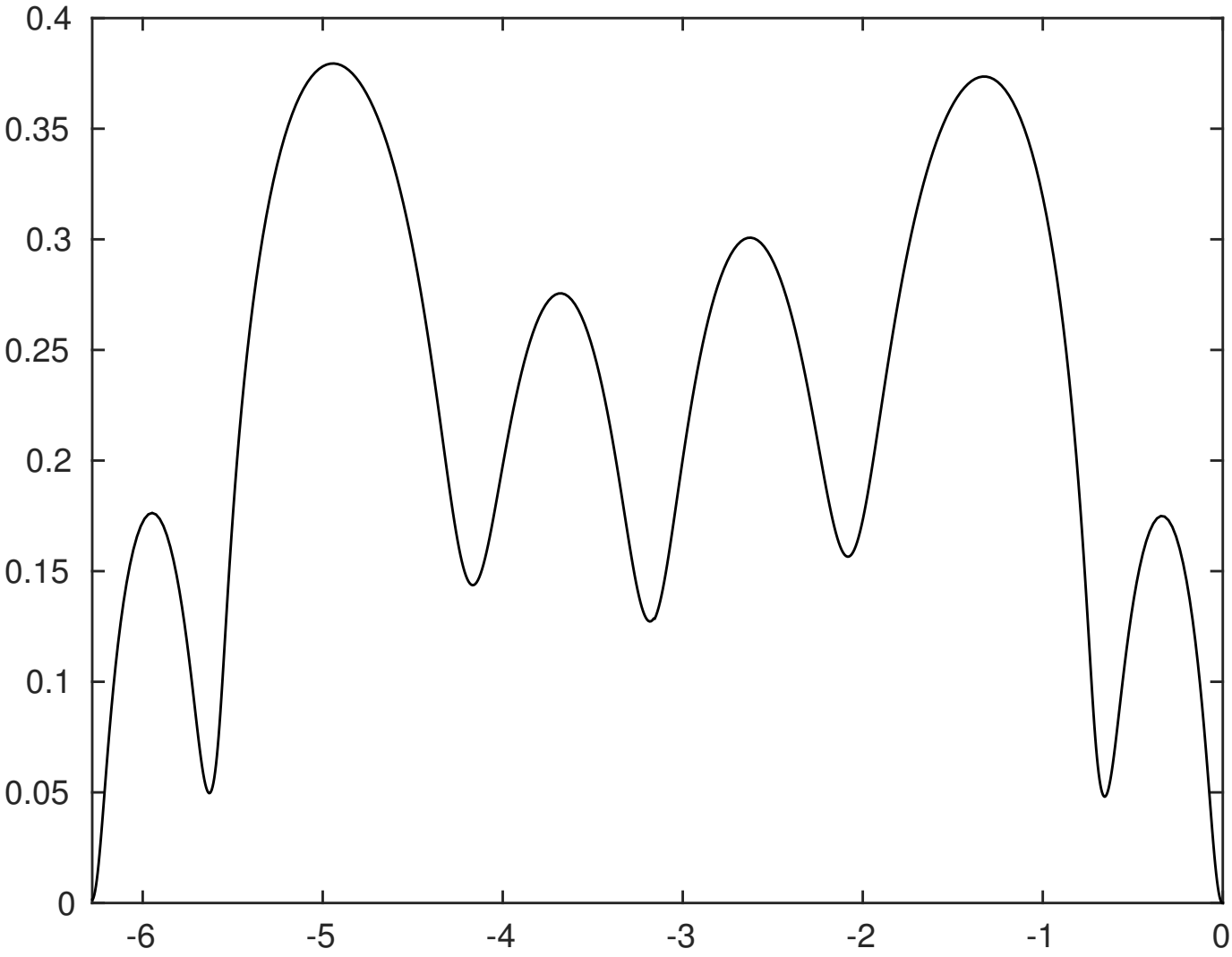
Non-symmetric waves

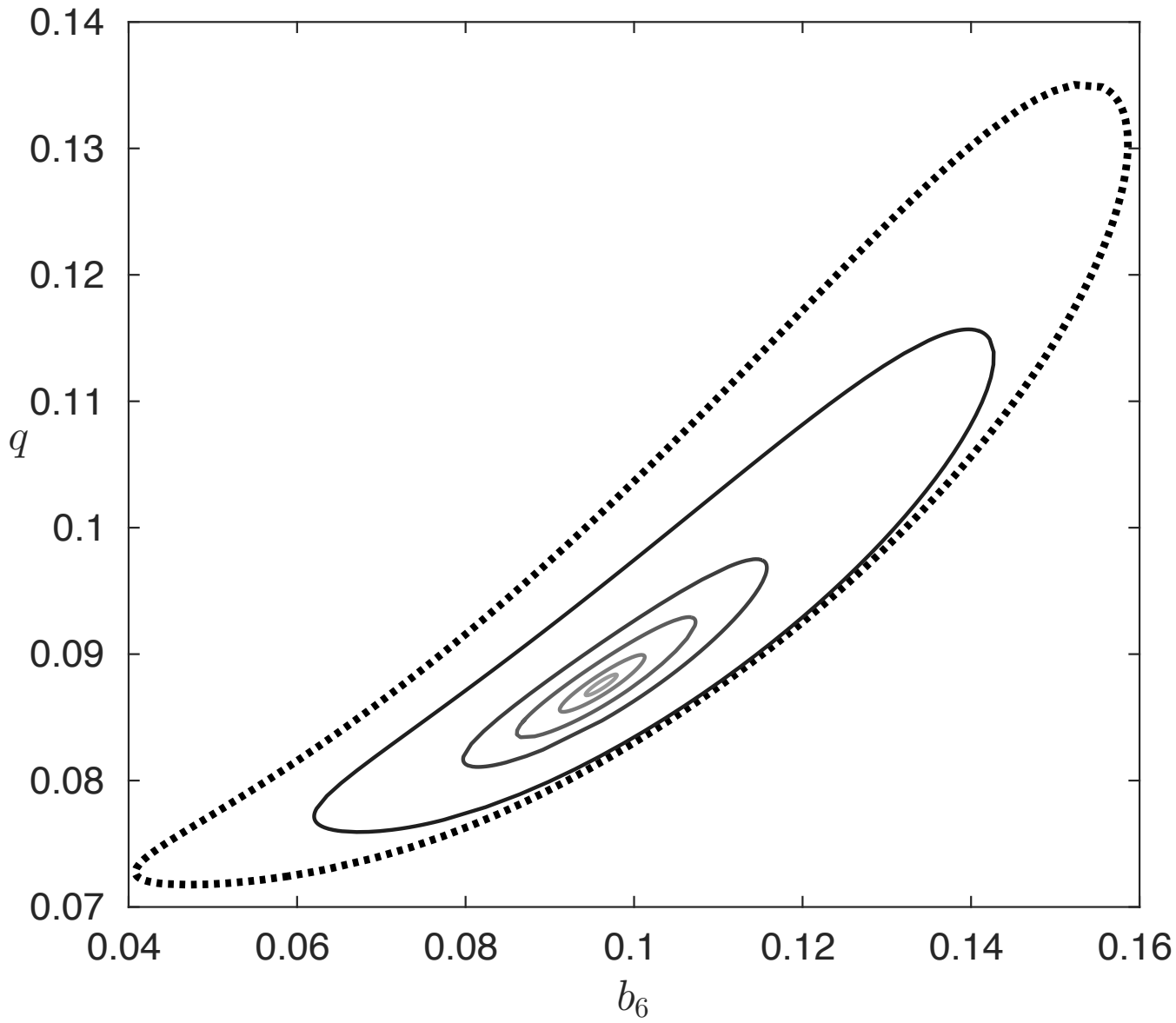


Non-symmetric waves



Non-symmetric waves



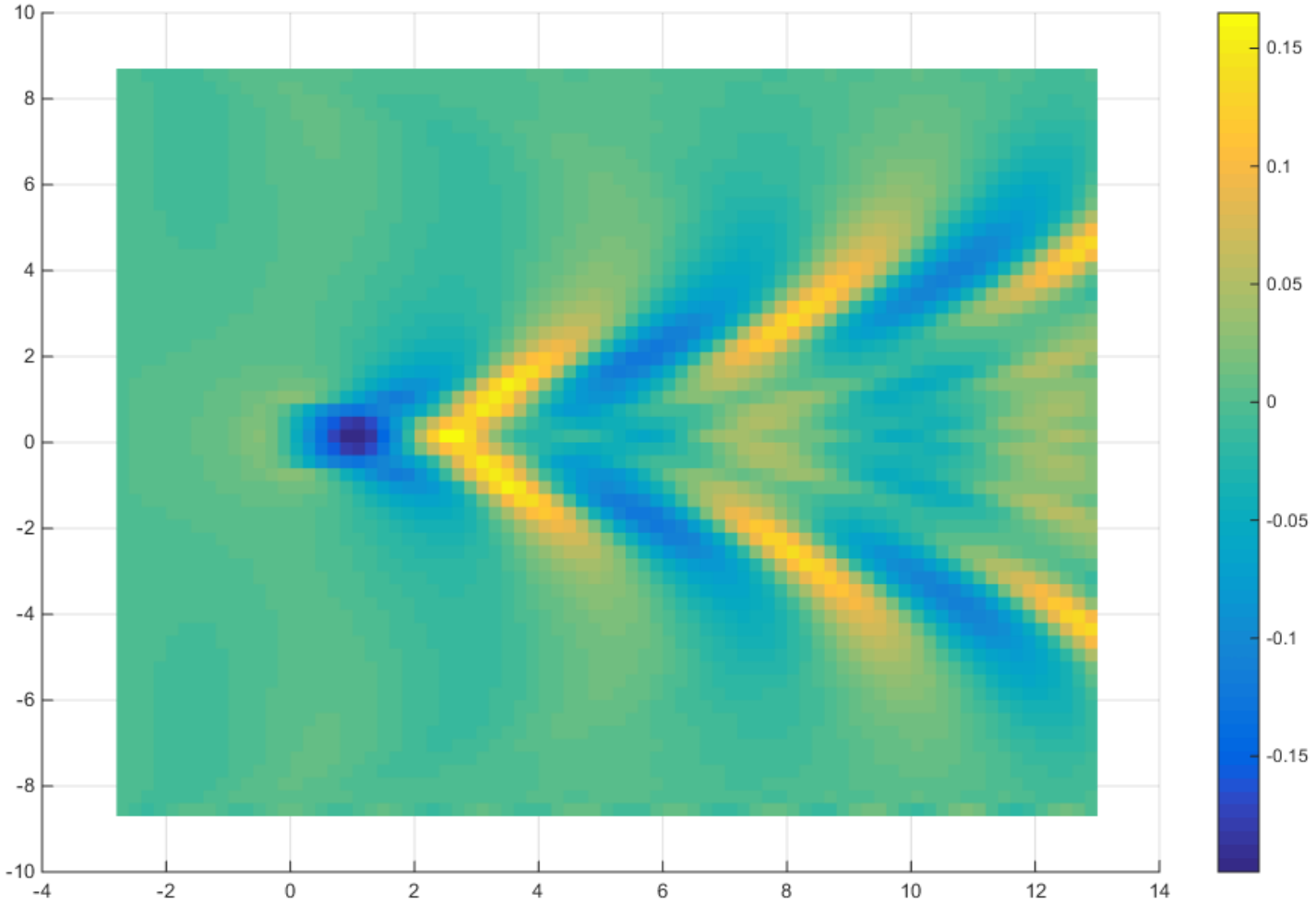

0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
q
0.04
0.06
0.08
0.1
0.12
0.14
0.16
b_6

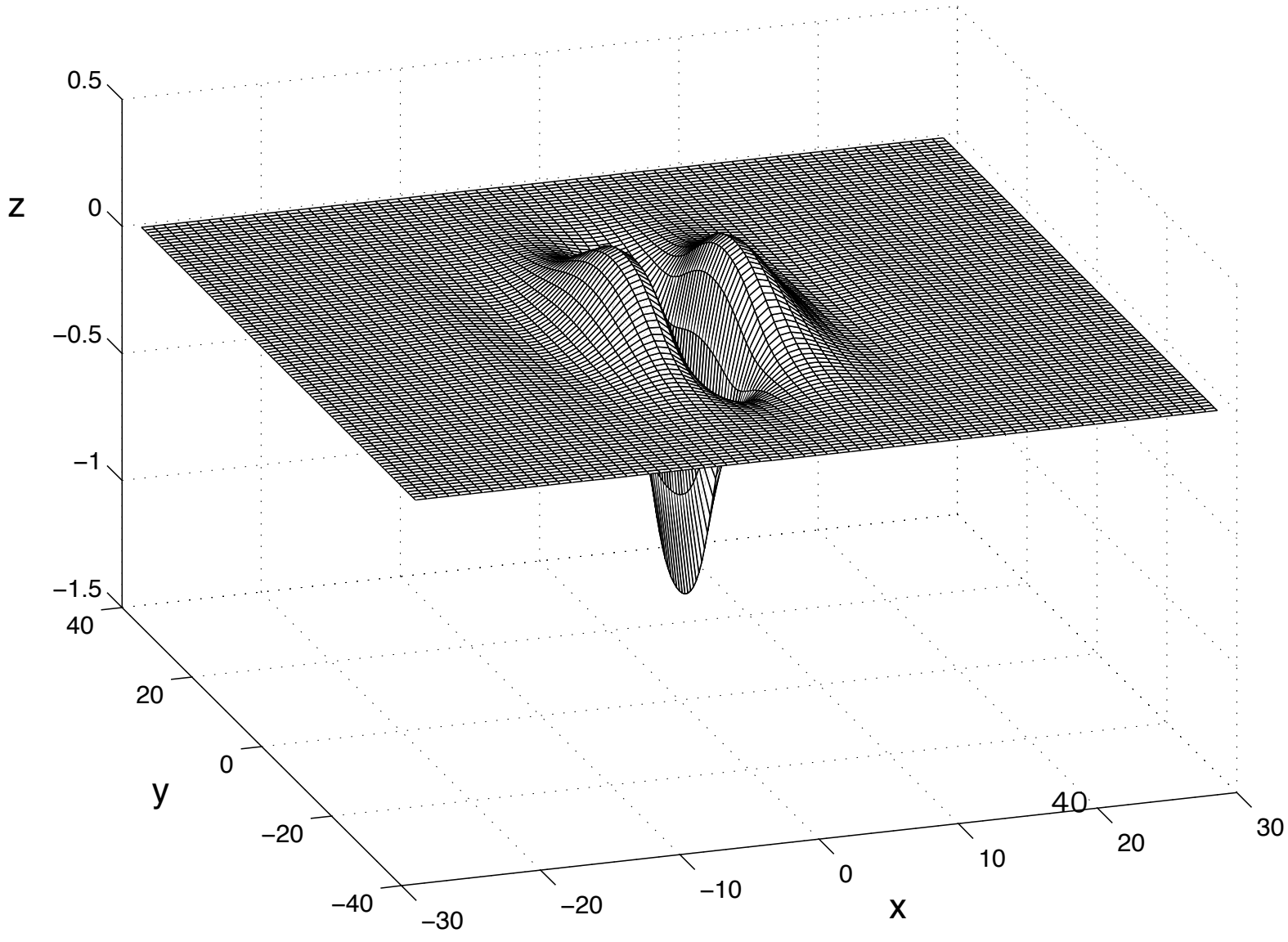
THREE-DIMENSIONAL FLOWS

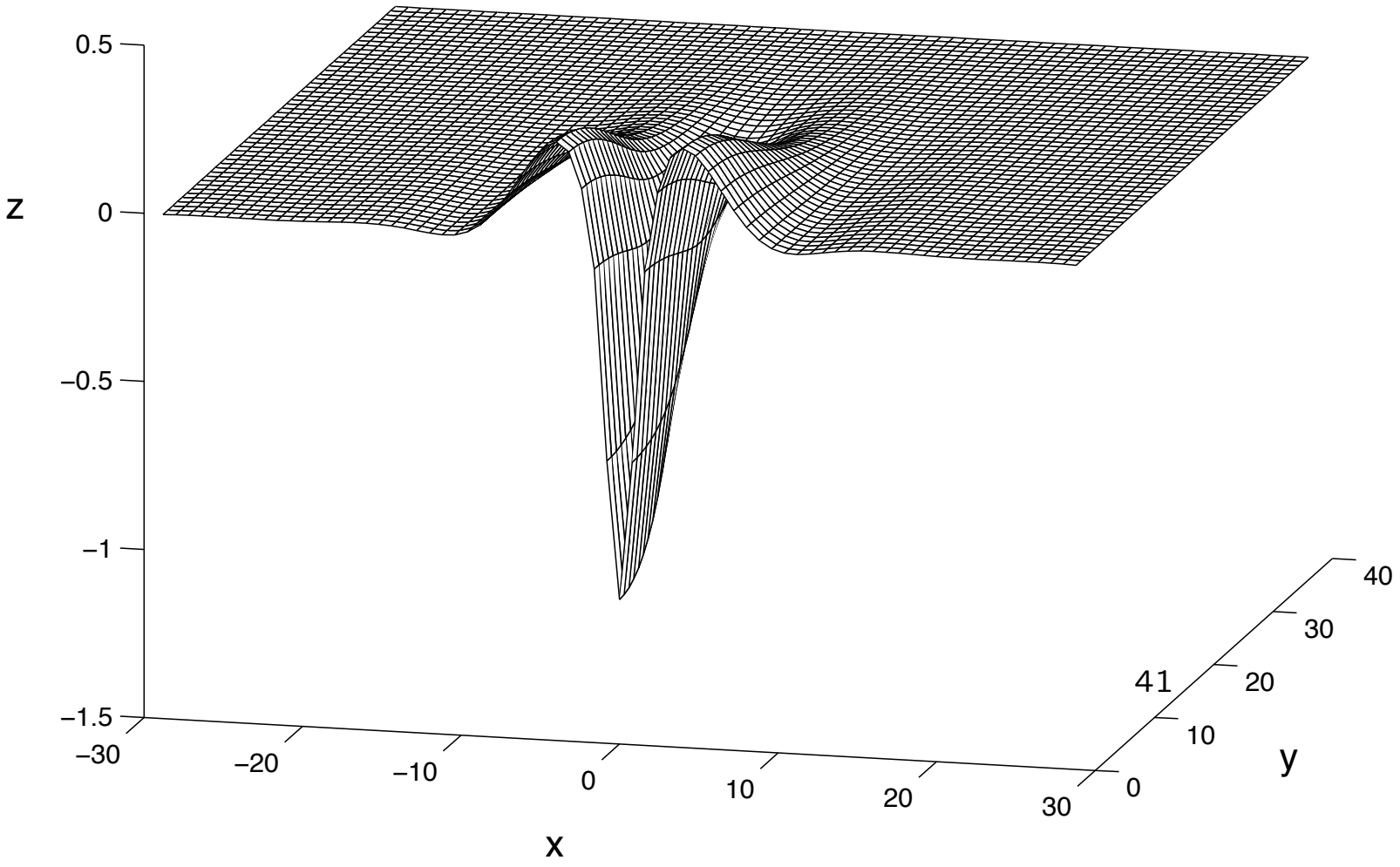
Use Green's theorem instead of Cauchy integral equation formula.

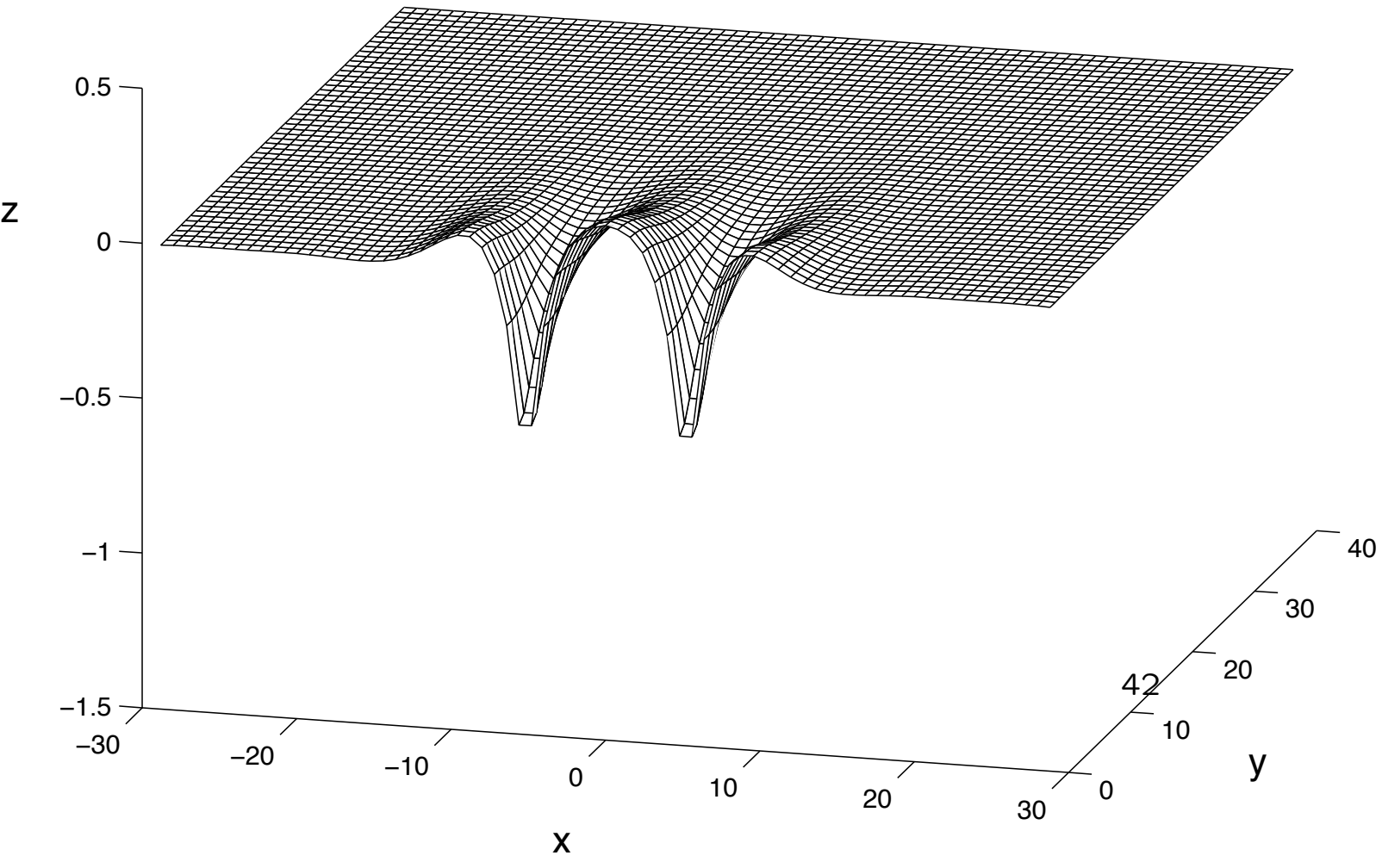
Emilian Parau, Mark Cooker, VdB

Olga Trichtchenko, Paul Milewski, Emilian Parau, VdB


10
8
6
4
2
0
-2
-4
-6
-8
-10
-4
-2
0
2
4
6
8
10
12
14
0.15
0.1
0.05
0
-0.05
-0.1
-0.15


0.5
0
−0.5
−1
−1.5
z
40
20
0
−20
−40
y
−30
−20
−10
0
10
20
30
x


0.5
0
−0.5
−1
−1.5
z
−30
−20
−10
0
10
20
30
x
0
10
20
30
40
y

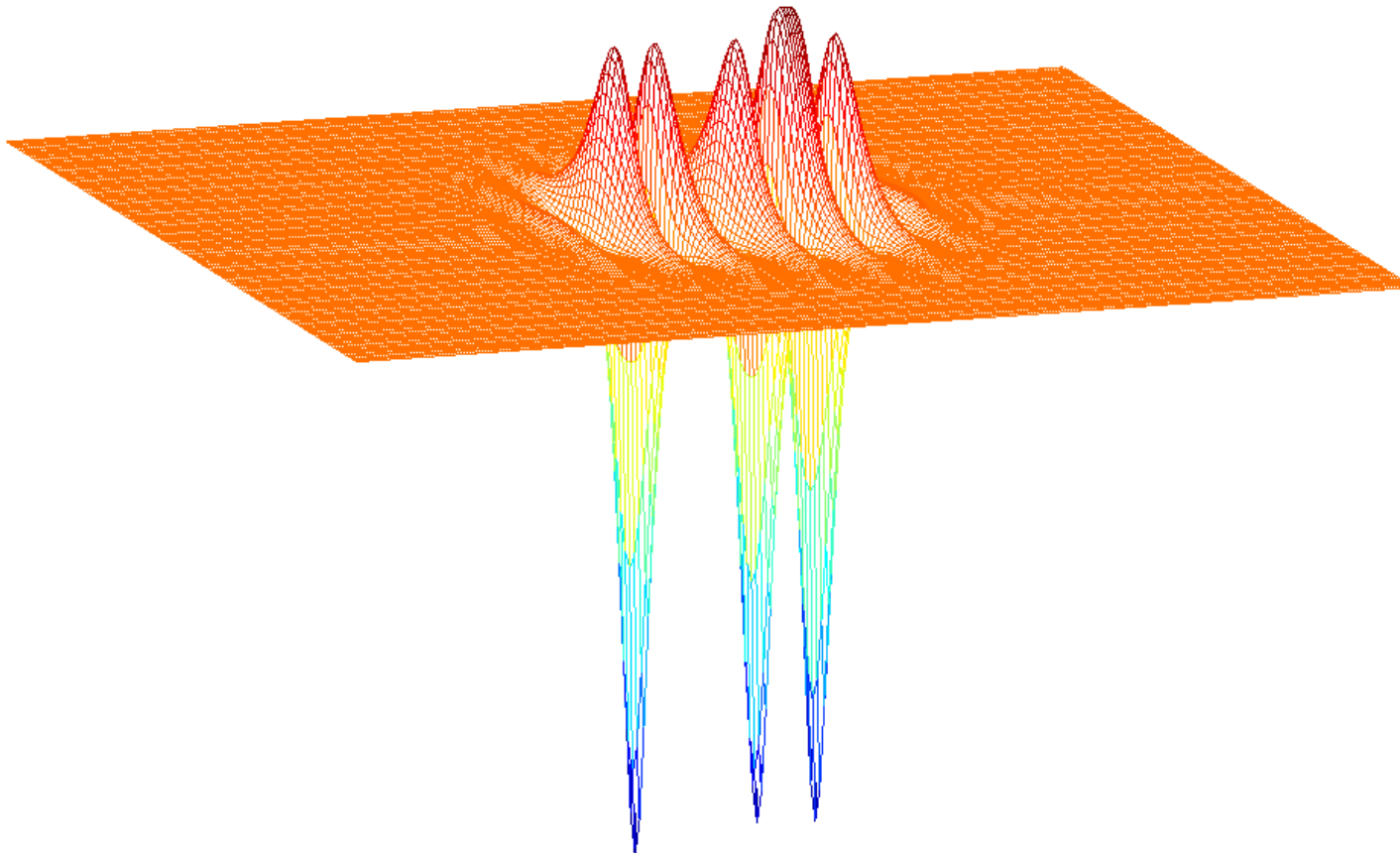

0.5
0
−0.5
−1
−1.5
z
−30
−20
−10
0
10
20
30
x
0
10
20
30
40
y

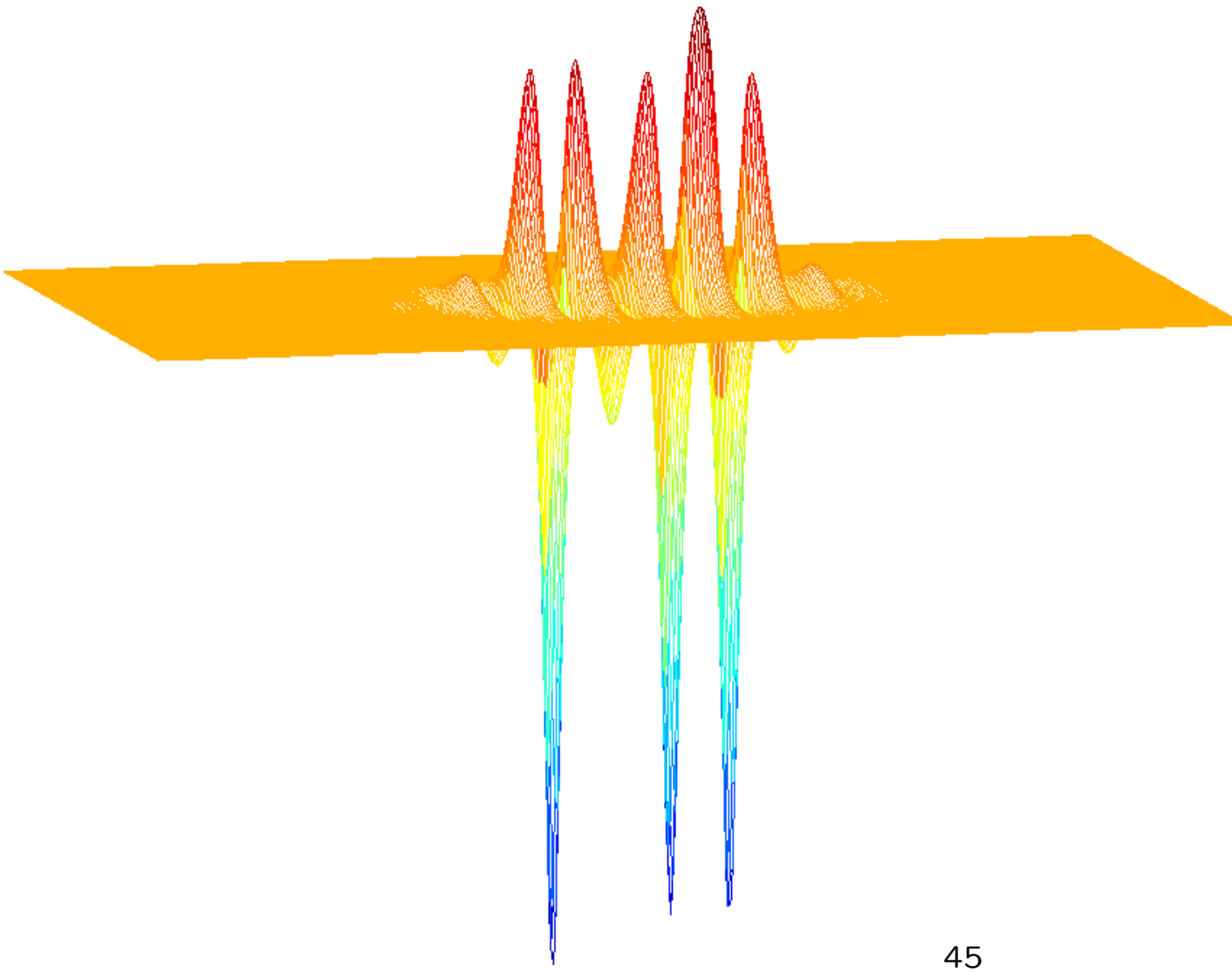
NON-SYMMETRIC 3D WAVES

Model: Akers and Milewski (2009)

$$u_t + \frac{\sqrt{2}}{2} u_x - \frac{\sqrt{2}}{4} H[u - u_{xx} - 2u_{yy}] + \alpha (u^2)_x = 0$$

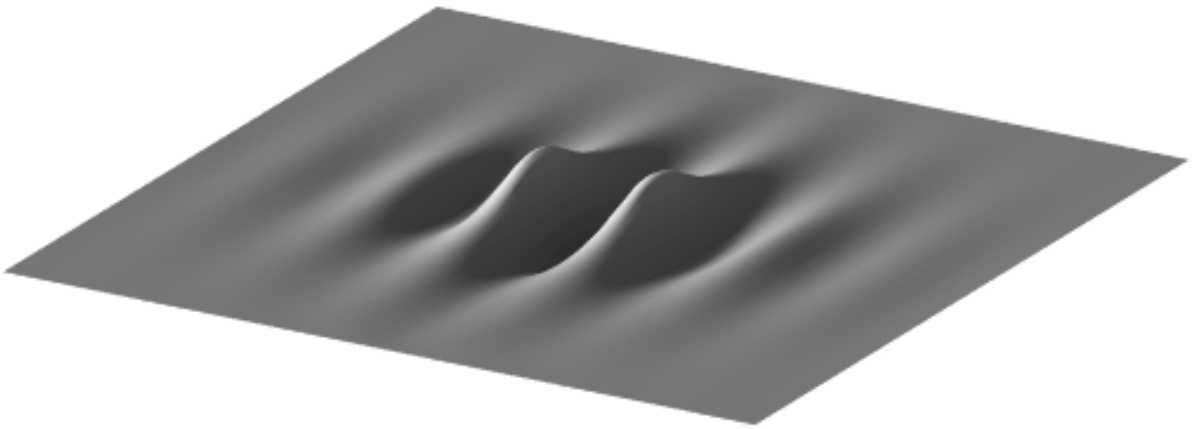
Zhan Wang

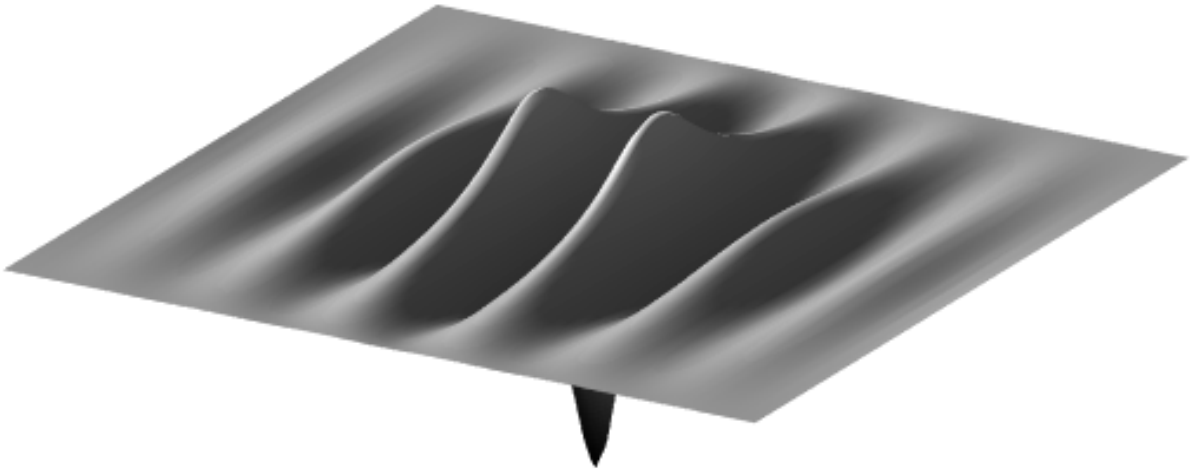


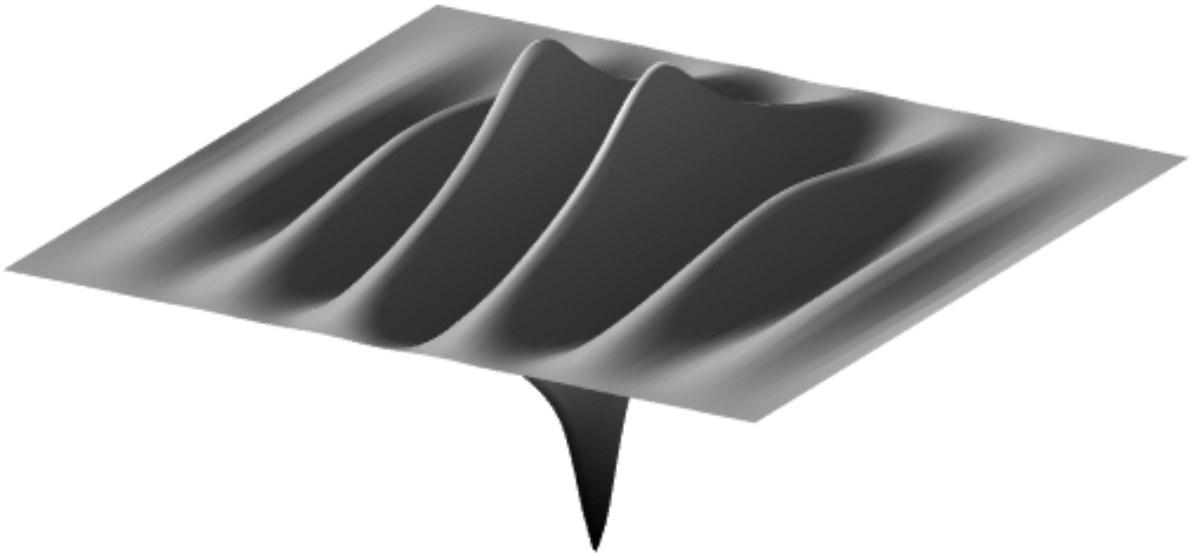


Three dimensional flexural waves

Olga....







Conclusions

New non-symmetric gravity-capillary waves for the Euler's equations in 2D (solitary waves)

New non-symmetric flexural waves for the Euler's equations in 2D (solitary waves)

New non-symmetric gravity-capillary waves for a model in 3D (solitary waves)

New non-symmetric generalised solitary waves in 2D

New non-symmetric periodic gravity-capillary waves in 2D

References

1. Wang Z. , Vanden-Broeck J.-M. and Milewski, PA., 2014, J. Fluid Mech. 759, R2

2. Gao T., Wang Z. and Vanden-Broeck J.-M., 2016, J. Fluid Mech. 788, 469-491

3. Gao T., Wang Z. and Vanden-Broeck J.-M. 2016, Proc. Roy,. Soc. A 472, No. 2194, p. 20160454

4. Gao T., Wang Z. and Vanden-Broeck J.-M. 2016, J. Fluid Mech. 811, 622-641